

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Monday 18 October 2021 – Afternoon**

**A Level Mathematics A**

**H240/03 Pure Mathematics and Mechanics**

**Time allowed: 2 hours**

**plus your additional time allowance**

**YOU MUST HAVE:**

**the Printed Answer Booklet sent with the standard paper or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre.**

**a scientific or graphical calculator**

**Insert for Questions 1, 4(a) and 10(a) (with this document)**

**READ INSTRUCTIONS OVERLEAF**



## **INSTRUCTIONS**

**Use black ink. You can use an HB pencil, but only for graphs and diagrams.**

**If you use the Printed Answer Booklet write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.**

**If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.**

**Answer ALL the questions.**

**Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.**

**Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.**

**The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.**

**Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.**

## **INFORMATION**

**The total mark for this paper is 100.**

**The marks for each question are shown in brackets [ ].**

## **ADVICE**

**Read each question carefully before you start your answer.**

## Formulae

### A Level Mathematics A (H240)

#### Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

#### Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \quad \text{for } |r| < 1$$

#### Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$$

$(n \in \mathbb{N}),$

where  ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$

$$(|x| < 1, n \in \mathbb{R})$$

## Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule  $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

## Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

## Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x) (f(x))^n dx = \frac{1}{n+1} (f(x))^{n+1} + c$$

Integration by parts  $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

## Small angle approximations

$\sin \theta \approx \theta$ ,  $\cos \theta \approx 1 - \frac{1}{2}\theta^2$ ,  $\tan \theta \approx \theta$  where  $\theta$  is measured in radians

## Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left( A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

## Numerical methods

Trapezium rule:

$$\int_a^b y \, dx \approx \frac{1}{2}h \left\{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \right\},$$

$$\text{where } h = \frac{b - a}{n}$$

The Newton-Raphson iteration for solving

$$f(x) = 0: \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

## Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B)$$

$$\text{OR } P(A | B) = \frac{P(A \cap B)}{P(B)}$$

## Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{OR} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

## The binomial distribution

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$ ,

mean of  $X$  is  $np$ , variance of  $X$  is  $np(1 - p)$

## Hypothesis test for the mean of a normal distribution

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

## Percentage points of the normal distribution

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

$p$	0.75	0.90	0.95	0.975	0.99	0.995
$z$	0.674	1.282	1.645	1.960	2.326	2.576

$p$	0.9975	0.999	0.9995
$z$	2.807	3.090	3.291

## Kinematics

### Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

### Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

## SECTION A: Pure Mathematics

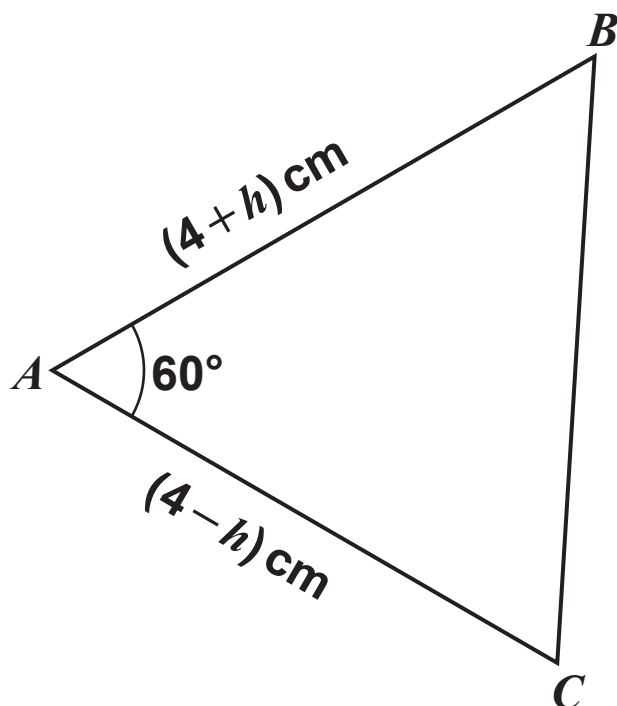
Answer ALL the questions.

- 1 Show in a sketch the region of the  $x$ - $y$  plane within which all three of the following inequalities hold.

$$y \geq x^2, \quad x + y \leq 2, \quad x \geq 0.$$

You should indicate the region for which the inequalities hold by labelling the region  $R$ . [3]





The diagram shows triangle  $ABC$  in which angle  $A$  is  $60^\circ$  and the lengths of  $AB$  and  $AC$  are  $(4 + h)$  cm and  $(4 - h)$  cm respectively.

(a) Show that the length of  $BC$  is  $p$  cm where  $p^2 = 16 + 3h^2$ . [2]

(b) Hence show that, when  $h$  is small,  
 $p \approx 4 + \lambda h^2 + \mu h^4$ , where  $\lambda$  and  $\mu$  are rational numbers whose values are to be determined. [4]

- 3 An arithmetic progression has first term 2 and common difference  $d$ , where  $d \neq 0$ . The first, third and thirteenth terms of this progression are also the first, second and third terms, respectively, of a geometric progression.**

**By determining  $d$ , show that the arithmetic progression is an increasing sequence. [5]**

- 4 (a) Sketch, on a single diagram, the following graphs.**

$$y = |x - 1|$$

$$y = \frac{k}{x}, \text{ where } k \text{ is a negative constant [2]}$$

- (b) Hence explain why the equation  $x|x - 1| = k$  has exactly one real root for any negative value of  $k$ . [1]**

- (c) Determine the real root of the equation  $x|x - 1| = -6$ . [2]**

- 5 A particle  $P$  moves along a straight line in such a way that at time  $t$  seconds  $P$  has velocity  $v \text{ m s}^{-1}$ , where

$$v = 12 \cos t + 5 \sin t.$$

- (a) Express  $v$  in the form  $R \cos(t - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . Give the value of  $\alpha$  correct to 4 significant figures. [3]
- (b) Hence find the two smallest positive values of  $t$  for which  $P$  is moving, in either direction, with a speed of  $3 \text{ m s}^{-1}$ . [3]

- 6 The equation  $6 \arcsin(2x - 1) - x^2 = 0$  has exactly one real root.

- (a) Show by calculation that the root lies between 0.5 and 0.6. [2]

In order to find the root, the iterative formula

$$x_{n+1} = p + q \sin(rx_n^2),$$

with initial value  $x_0 = 0.5$ , is to be used.

- (b) Determine the values of the constants  $p$ ,  $q$  and  $r$ . [2]
- (c) Hence find the root correct to 4 significant figures. Show the result of each step of the iteration process. [2]

- 7 A curve  $C$  in the  $x$ - $y$  plane has the property that the gradient of the tangent at the point  $P(x, y)$  is three times the gradient of the line joining the point  $(3, 2)$  to  $P$ .**

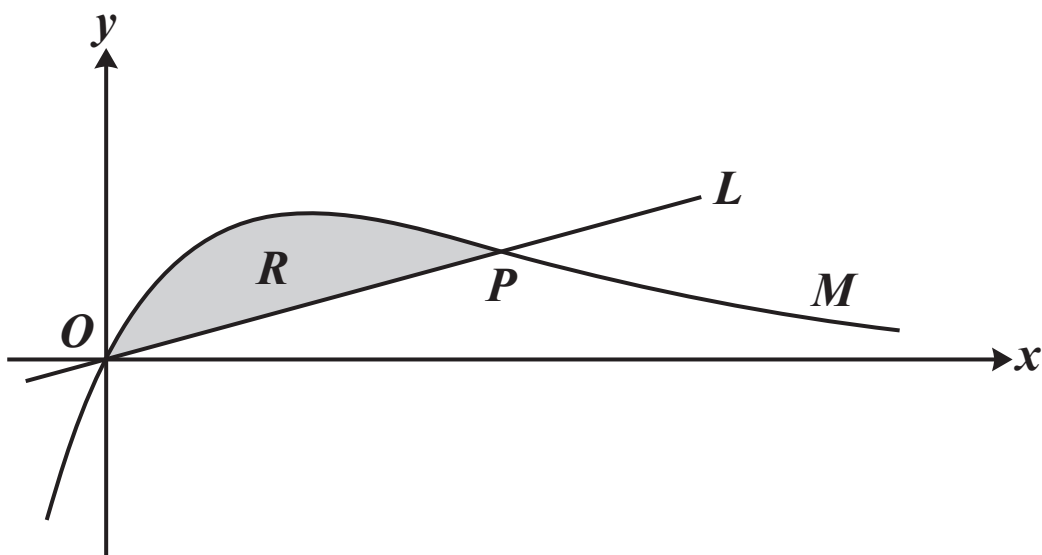
**(a) Express this property in the form of a differential equation. [2]**

**It is given that  $C$  passes through the point  $(4, 3)$  and that  $x > 3$  and  $y > 2$  at all points on  $C$ .**

**(b) Determine the equation of  $C$  giving your answer in the form  $y = f(x)$ . [4]**

**The curve  $C$  may be obtained by a transformation of part of the curve  $y = x^3$ .**

**(c) Describe fully this transformation. [2]**



The diagram shows the curve  $M$  with equation  $y = xe^{-2x}$ .

- (a) Show that  $M$  has a point of inflection at the point  $P$  where  $x = 1$ . [5]

The line  $L$  passes through the origin  $O$  and the point  $P$ . The shaded region  $R$  is enclosed by the curve  $M$  and the line  $L$ .

- (b) Show that the area of  $R$  is given by

$$\frac{1}{4}(a + be^{-2}),$$

where  $a$  and  $b$  are integers to be determined. [6]

## SECTION B: Mechanics

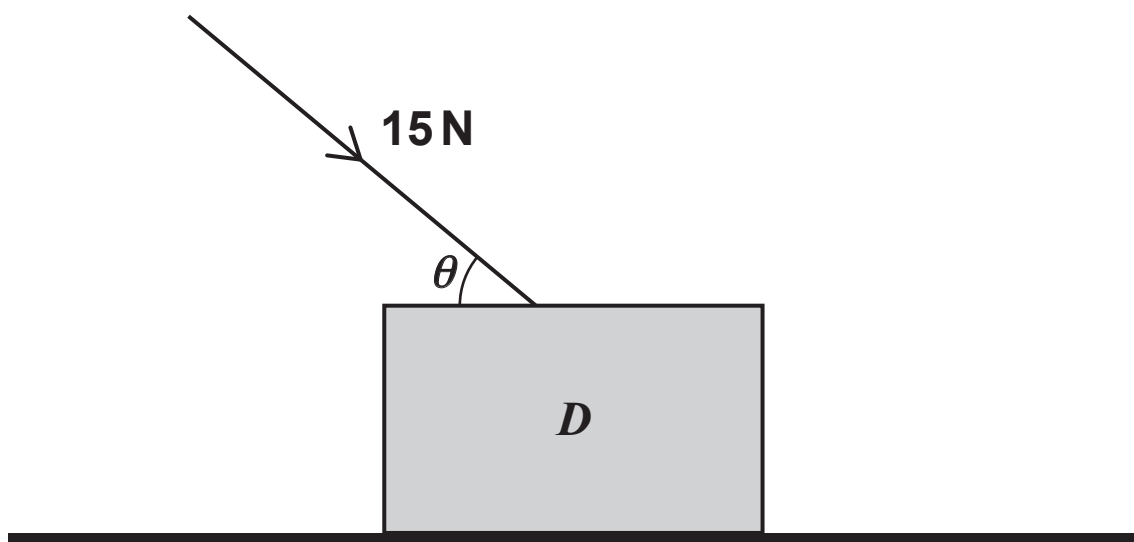
Answer ALL the questions.

- 9 There are three checkpoints,  $A$ ,  $B$  and  $C$ , in that order, on a straight horizontal road. A car travels along the road, in the direction from  $A$  to  $C$ , with constant acceleration. The car takes 20 s to travel from  $B$  to  $C$ . The speed of the car at  $B$  is  $14 \text{ m s}^{-1}$  and the speed of the car at  $C$  is  $18 \text{ m s}^{-1}$ .

(a) Find the acceleration of the car. [1]

It is given that the distance between  $A$  and  $B$  is 330 m.

(b) Determine the speed of the car at  $A$ . [2]



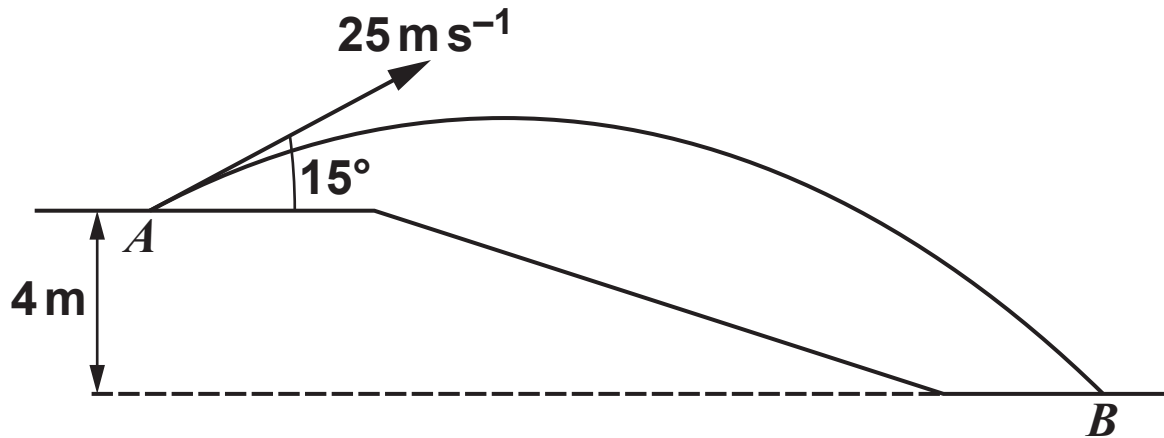
A block  $D$  of weight  $50\text{ N}$  lies at rest in equilibrium on a fixed rough horizontal surface. A force of magnitude  $15\text{ N}$  is applied to  $D$  at an angle  $\theta$  to the horizontal (see diagram).

- (a) Complete the diagram in the Printed Answer Booklet or in the insert showing all the forces acting on  $D$ . [1]

It is given that  $D$  remains at rest and the coefficient of friction between  $D$  and the surface is  $0.2$ .

- (b) Show that

$$15 \cos \theta - 3 \sin \theta \leq 10. \quad [5]$$



A golfer hits a ball from a point  $A$  with a speed of  $25\text{ m s}^{-1}$  at an angle of  $15^\circ$  above the horizontal. While the ball is in the air, it is modelled as a particle moving under the influence of gravity. Take the acceleration due to gravity to be  $10\text{ m s}^{-2}$ .

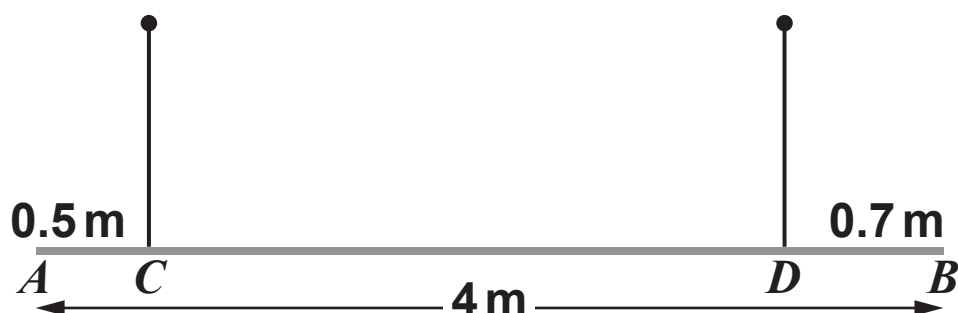
The ball first lands at a point  $B$  which is  $4\text{ m}$  below the level of  $A$  (see diagram).

- (a) Determine the time taken for the ball to travel from  $A$  to  $B$ . [3]
- (b) Determine the horizontal distance of  $B$  from  $A$ . [2]
- (c) Determine the direction of motion of the ball  $1.5$  seconds after the golfer hits the ball. [4]

The horizontal distance from  $A$  to  $B$  is found to be greater than the answer to part (b).

- (d) State one factor that could account for this difference. [1]





A beam,  $AB$ , has length 4 m and mass 20 kg. The beam is suspended horizontally by two vertical ropes. One rope is attached to the beam at  $C$ , where  $AC = 0.5$  m. The other rope is attached to the beam at  $D$ , where  $DB = 0.7$  m (see diagram).

The beam is modelled as a non-uniform rod and the ropes as light inextensible strings.

It is given that the tension in the rope at  $C$  is three times the tension in the rope at  $D$ .

- (a) Determine the distance of the centre of mass of the beam from  $A$ . [5]

A particle of mass  $m$  kg is now placed on the beam at a point where the magnitude of the moment of the particle's weight about  $C$  is  $3.5mg$  N m. The beam remains horizontal and in equilibrium.

- (b) Determine the largest possible value of  $m$ . [2]

- 13** In this question the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are in the directions east and north respectively.

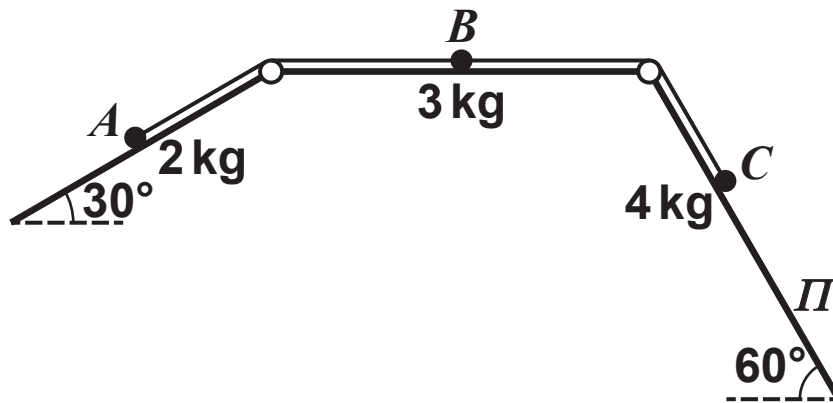
At time  $t$  seconds, where  $t \geq 0$ , a particle  $P$  of mass  $2\text{ kg}$  is moving on a smooth horizontal surface under the action of a constant horizontal force  $(-8\mathbf{i} - 54\mathbf{j})\text{ N}$  and a variable horizontal force  $(4t\mathbf{i} + 6(2t - 1)^2\mathbf{j})\text{ N}$ .

- (a) Determine the value of  $t$  when the forces acting on  $P$  are in equilibrium. [2]

It is given that  $P$  is at rest when  $t = 0$ .

- (b) Determine the speed of  $P$  at the instant when  $P$  is moving due north. [6]
- (c) Determine the distance between the positions of  $P$  when  $t = 0$  and  $t = 3$ . [5]

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One end of a light inextensible string is attached to a particle  $A$  of mass 2 kg. The other end of the string is attached to a second particle  $B$  of mass 3 kg. Particle  $A$  is in contact with a smooth plane inclined at  $30^\circ$  to the horizontal and particle  $B$  is in contact with a rough horizontal plane.

A second light inextensible string is attached to  $B$ . The other end of this second string is attached to a third particle  $C$  of mass 4 kg. Particle  $C$  is in contact with a smooth plane  $\Pi$  inclined at an angle of  $60^\circ$  to the horizontal.

Both strings are taut and pass over small smooth pulleys that are at the tops of the inclined planes. The parts of the strings from  $A$  to the pulley, and from  $C$  to the pulley, are parallel to lines of greatest slope of the corresponding planes (see diagram).

The coefficient of friction between  $B$  and the horizontal plane is  $\mu$ . The system is released from rest and in the subsequent motion  $C$  moves down  $\Pi$  with acceleration  $a \text{ m s}^{-2}$ .

- (a) By considering an equation involving  $\mu$ ,  $a$  and  $g$  show that  $a < \frac{1}{9}g(2\sqrt{3} - 1)$ . [7]
- (b) Given that  $a = \frac{1}{9}g$ , determine the magnitude of the contact force between  $B$  and the horizontal plane. Give your answer correct to 3 significant figures. [4]

**END OF QUESTION PAPER**

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